Problem 1.58

A rectangular piece of aluminum is 7.60 ± 0.01 cm long and 1.90 ± 0.01 cm wide. (a) Find the area of the rectangle and the uncertainty in the area. (b) Verify that the fractional uncertainty in the area is equal to the sum of the fractional uncertainties in the length and in the width. (This is a general result; see Challenge Problem 1.98.)

Solution

Part (a)

The area of this aluminum is the length times the width,

$$A = lw,$$

where

$$\begin{aligned} l &= 7.60 \pm 0.01 \text{ cm} \\ w &= 1.90 \pm 0.01 \text{ cm}. \end{aligned}$$

Plugging in the numbers gives

$$A = (7.60 \pm 0.01 \text{ cm})(1.90 \pm 0.01 \text{ cm}).$$

The lower and upper bounds for the area are

$$\begin{aligned} \text{Lower Bound}: &\quad A = (7.59 \text{ cm})(1.89 \text{ cm}) \approx 14.3 \text{ cm}^2 \\ \text{Upper Bound}: &\quad A = (7.61 \text{ cm})(1.91 \text{ cm}) \approx 14.5 \text{ cm}^2. \end{aligned}$$

Take the average of these two bounds.

$$\frac{14.3 + 14.5}{2} = 14.4$$

Both bounds can be covered with an uncertainty of 0.1. Therefore, the area is

$$A = (14.4 \pm 0.1) \text{ cm}^2.$$

Part (b)

The fractional uncertainty in the area is

$$\frac{\Delta A}{A} \approx \frac{0.1}{14.4} \approx 0.00694444,$$

and the fractional uncertainties in the length and width are

$$\left.\begin{aligned} \frac{\Delta l}{l} &= \frac{0.01}{7.60} \approx 0.00131579 \\ \frac{\Delta w}{w} &= \frac{0.01}{1.90} \approx 0.00526316 \end{aligned}\right\} \quad \Rightarrow \quad \frac{\Delta l}{l} + \frac{\Delta w}{w} \approx 0.00657895.$$